November 2022 Problem of the Month

Let *A* be the set of positive integers that contain only the digits 1 or 2. We then define the following sum.

$$S = \sum_{n \in A} \frac{1}{n} = \frac{1}{1} + \frac{1}{2} + \frac{1}{11} + \frac{1}{12} + \frac{1}{21} + \frac{1}{22} + \frac{1}{111} + \cdots$$

a) How many elements of set A contain exactly n digits? Your answer should be in terms of n.

- b) Show that $S \leq 3$. You may want to use part (a).
- c) Find both upper and lower bounds as accurate as you can for S.

Please email solutions to Dr London at <u>slondon@luc.edu</u> in PDF form by 11:59 pm on November 30.



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